

Electrical and optical properties of materials

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Part 1. Conductivity: from insulators to superconductors

$$\sigma = j/E, \quad (1.1)$$

$$-eE = m_e \frac{dv}{dt} \quad (1.2)$$

$$-eE\tau = m_e v_d \quad (1.3)$$

$$\tau = \lambda_s/v_F \quad (1.4)$$

$$j = -nev_d = \frac{ne^2 E \lambda_s}{m_e v_F} \quad (1.5)$$

$$\sigma = \frac{ne^2 \lambda_s}{m_e v_F}, \quad (1.6)$$

$$\sigma \propto n \propto \exp(-E_g/2k_B T) \quad (1.7)$$

$$G_p = U_p - TS_p \quad (1.8)$$

$$G_a = G_p + n\phi - 2k_B T \ln[(N+n)!/N!n!] - Tn\Delta S_{freq} \quad (1.9)$$

$$\frac{\partial G_a}{\partial n} = \phi - 2k_B T [\ln(N+n) - \ln n] - T\Delta S_{freq} = 0 \quad (1.10)$$

$$n = N \exp\left(\frac{\Delta S_{freq}}{2k_B} - \frac{\phi}{2k_B T}\right) \quad (1.11)$$

$$S = k_B [1 + \ln(k_B T/\hbar\omega)] \quad (1.12)$$

$$S_p = 6Nk_B + 6Nk_B \ln(k_B T/\hbar\omega) \quad (1.13)$$

$$S_a = (6N - 6nZ) \left(k_B + k_B \ln \frac{k_B T}{\hbar\omega}\right) + 6nZ \left(k_B + k_B \ln \frac{k_B T}{\hbar\omega'}\right) \quad (1.14)$$

1. Conductivity: from insulators to superconductors

$$\Delta S_{freq} = \frac{Sa - Sp}{n} = 6Zk_B \ln \frac{\omega}{\omega'} \quad (1.15)$$

$$\frac{n}{N} = \left(\frac{\omega}{\omega'}\right)^{3Z} \exp\left(\frac{-\phi}{2k_B T}\right) = C \exp\left(\frac{-\phi}{2k_B T}\right) \quad (1.16)$$

$$n = C\sqrt{NN_i} \exp\left(\frac{-\phi}{2k_B T}\right) \quad (1.17)$$

$$J = -\mathcal{D} \frac{dN}{dx} \quad (1.18)$$

$$\mathcal{D} = \frac{v_d k_B T}{qE} \quad (1.19)$$

$$\sigma = \left(\frac{n_c e^2}{k_B T}\right) \mathcal{D} \quad (1.20)$$

$$p = \nu \exp\left(\frac{-\epsilon_i}{k_B T}\right) \quad (1.21)$$

$$\mathcal{D} = \frac{1}{3} a^2 \frac{n}{N} p, \quad (1.22)$$

$$\mathcal{D} = \frac{1}{3} a^2 \nu \exp\left(\frac{-\epsilon_i}{k_B T}\right) \left[\frac{n_{ext}}{N} + C \exp\left(\frac{-\phi}{2k_B T}\right) \right] \quad (1.23)$$

$$\ln \mathcal{D} = \ln \frac{\nu a^2}{3} - \frac{\epsilon_i}{k_B T} + \ln \left[\frac{n_{ext}}{N} + C \exp\left(\frac{-\phi}{2k_B T}\right) \right] \quad (1.24)$$

$$\ln \mathcal{D} = -\frac{2\epsilon_i + \phi}{2k_B T} + \left(\ln \frac{\nu a^2}{3} + \ln C \right) \quad (1.25)$$

$$\ln \mathcal{D} = -\frac{\epsilon_i}{k_B T} + \left(\ln \frac{\nu a^2}{3} + \ln \frac{n_{ext}}{N} \right) \quad (1.26)$$

$$p_+ = \frac{\nu}{3} \exp \left[- \left(\epsilon_i - \frac{eaE}{2} \right) / k_B T \right] \quad (1.27)$$

$$p_- = \frac{\nu}{3} \exp \left[- \left(\epsilon_i + \frac{eaE}{2} \right) / k_B T \right] \quad (1.28)$$

$$I_{channel} = e \frac{n}{N} (p_+ - p_-) \quad (1.29)$$

$$j = \frac{2}{a^2} e \frac{n}{N} (p_+ - p_-) \quad (1.30)$$

$$j = \frac{2}{a^2} e \frac{n}{N} \frac{\nu}{3} \exp\left(\frac{-\epsilon_i}{k_B T}\right) \left[\exp\left(\frac{eaE}{2k_B T}\right) - \exp\left(\frac{-eaE}{2k_B T}\right) \right] \quad (1.31)$$

$$\sigma = \frac{2e^2 \nu}{3ak_B T} \frac{n}{N} \exp\left(\frac{-\epsilon_i}{k_B T}\right) \quad (1.32)$$

$$\mathcal{D} = \frac{a^2}{3} \frac{n}{N} \nu \exp\left(\frac{-\epsilon_i}{k_B T}\right) \quad (1.33)$$

$$\xi = -\frac{\Delta G}{F_z} \quad (1.34)$$

$$\Delta G = -RT \ln\left(\frac{a_1}{a_2}\right) \quad (1.35)$$

$$\xi = \frac{RT}{F_z} \ln\left(\frac{a_1}{a_2}\right) \quad (1.36)$$

$$\Delta G_{super} = \frac{1}{2} \mu H_c^2 \quad (1.37)$$

$$\Delta\phi = \frac{q\Phi}{\hbar} \quad (1.38)$$

$$\Delta\phi = \frac{q\Phi}{\hbar} = 2N\pi \quad \text{and thus,} \quad \Phi = \frac{Nh}{q} = N\Phi_0 \quad (1.39)$$

$$\Phi_0 = \frac{h}{2e} = 2 \times 10^{-15} \text{ Weber} \quad (1.40)$$

$$I_c = 2\pi r H_c \quad (1.41)$$

$$F_z = -nev_x N\Phi_0 \quad (1.42)$$

$$F_z = j_x N\Phi_0 \quad (1.43)$$

$$F_{pinning} = j_{crit} \Phi_0 \quad (1.44)$$