

Electrical and optical properties of materials

John JL Morton

Part 2: Dielectric properties of materials

$$D = \epsilon_0 E + P \quad (2.1)$$

$$D = \epsilon_0 E + \chi_e \epsilon_0 E = \epsilon_0 (1 + \chi_e) E = \epsilon_0 \epsilon_r E \quad (2.2)$$

$$\epsilon_r = \frac{P}{\epsilon_0 E} + 1 \quad (2.3)$$

$$I = I_R + I_C = \frac{V_0 \sin \omega t}{R} + CV_0 \omega \cos \omega t \quad (2.4)$$

$$\tan \delta = \frac{I_R}{I_C} = \frac{V_0/R}{CV_0 \omega} = \frac{1}{\omega CR} \quad (2.5)$$

$$W = \frac{1}{T} \int_0^T VI = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} V_0 \sin \omega t \left(\frac{V_0 \sin \omega t}{R} + CV_0 \omega \cos \omega t \right) dt \quad (2.6)$$

$$W = \frac{\omega V_0^2}{2\pi} \int_0^{2\pi/\omega} \left(\frac{1 - \cos 2\omega t}{2R} + C\omega \sin \omega t \cos \omega t \right) dt \quad (2.7)$$

$$W = \frac{V_0^2}{2R} = \frac{1}{2} \omega V_0^2 C \tan \delta = \frac{1}{2} \omega V_0^2 C_0 \epsilon_r \tan \delta \quad (2.8)$$

$$Z = \frac{1}{i\omega C} = \frac{1}{i\omega \epsilon_r C_0} \quad (2.9)$$

$$Z = \frac{1}{i\omega C_0 [\text{Re}(\epsilon_r) + i\text{Im}(\epsilon_r)]} = \frac{\text{Re}(\epsilon_r)}{i\omega C_0 |\epsilon_r|^2} - \frac{\text{Im}(\epsilon_r)}{\omega C_0 |\epsilon_r|^2} \quad (2.10)$$

$$C' = \frac{C_0 |\epsilon_r|^2}{\text{Re}(\epsilon_r)}, \quad R = \frac{\text{Im}(\epsilon_r)}{\omega C_0 |\epsilon_r|^2}, \quad \text{and hence} \quad \tan \delta = \frac{1}{\omega CR} = \frac{\text{Im}(\epsilon_r)}{\text{Re}(\epsilon_r)} \quad (2.11)$$

$$P = n\alpha E_{loc}, \quad (2.12)$$

$$\delta E_r = \frac{P \cos \theta}{4\pi \epsilon_0 r^2} \delta A \quad (2.13)$$

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$$E_s = \oiint_{\text{surface}} \delta E_r \cos \theta \quad (2.14)$$

$$E_s = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{P \cos^2 \theta}{4\pi\epsilon_0 r^2} r^2 \sin \theta \delta\theta \delta\phi \quad (2.15)$$

$$E_s = \int_{\theta=0}^{\pi} \frac{P \cos^2 \theta}{4\pi\epsilon_0 r^2} 2\pi r^2 \sin \theta \delta\theta \quad (2.16)$$

$$E_s = \int_{\theta=0}^{\pi} \frac{P \cos^2 \theta}{2\epsilon_0} \sin \theta \delta\theta \quad (2.17)$$

$$E_s = \frac{P}{2\epsilon_0} \left[\frac{\cos^3 \theta}{3} \right]_0^{\pi} \quad (2.18)$$

$$E_s = \frac{P}{3\epsilon_0} \quad (2.19)$$

$$E_{loc} = E + \frac{P}{3\epsilon_0} \quad (2.20)$$

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{n\alpha}{3\epsilon_0} \quad (2.21)$$

$$\delta n = C \exp\left(\frac{-U}{k_B T}\right) \delta U \quad (2.22)$$

$$P = n \frac{\int p \cos \theta dn}{\int dn} \quad (2.23)$$

$$P = n \frac{\int_0^{\pi} p \cos \theta C \exp\left(\frac{pE_{loc} \cos \theta}{k_B T}\right) dU}{\int_0^{\pi} C \exp\left(\frac{pE_{loc} \cos \theta}{k_B T}\right) dU} \quad (2.24)$$

$$P = n \frac{\int_0^{\pi} p \cos \theta C \exp\left(\frac{pE_{loc} \cos \theta}{k_B T}\right) pE_{loc} \sin \theta d\theta}{\int_0^{\pi} C \exp\left(\frac{pE_{loc} \cos \theta}{k_B T}\right) pE_{loc} \sin \theta d\theta} \quad (2.25)$$

$$P = n \frac{\int_1^{-1} x \exp(xy) dx}{\int_1^{-1} \exp(xy) dx} \quad (2.26)$$

$$P = np \left(\coth y - \frac{1}{y} \right) = npL(y) \quad (2.27)$$

$$P = \frac{np^2}{3k_B T} E_{loc} \quad (2.28)$$

$$m \left(\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x \right) = qE_0 \exp(i\omega t) \quad (2.29)$$

$$x = x_0 \exp(i\omega t) = \frac{q}{m} \frac{1}{(\omega_0^2 - \omega^2) + i\omega\gamma} E_0 \exp(i\omega t) \quad (2.30)$$

$$P = np = nqx = \frac{nq^2}{m} \frac{1}{(\omega_0^2 - \omega^2) + i\omega\gamma} E_0 \exp(i\omega t) \quad (2.31)$$

$$\alpha = \frac{q^2}{m} \frac{1}{(\omega_0^2 - \omega^2) + i\omega\gamma} \quad (2.32)$$

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{nq^2}{3m\epsilon_0} \frac{1}{(\omega_0^2 - \omega^2) + i\omega\gamma} \quad (2.33)$$

$$\frac{n\alpha}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2} \approx \frac{\epsilon_r - 1}{3} \quad (2.34)$$

$$\epsilon_r - 1 = \frac{nq^2}{m\epsilon_0} \frac{1}{(\omega_0^2 - \omega^2) + i\omega\gamma} \quad (2.35)$$

$$\text{Re}(\epsilon_r) = 1 + \frac{nq^2}{m\epsilon_0} \frac{(\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + \omega^2\gamma^2} \quad (2.36)$$

$$\text{Im}(\epsilon_r) = -\frac{nq^2}{m\epsilon_0} \frac{\omega\gamma}{(\omega_0^2 - \omega^2)^2 + \omega^2\gamma^2} \quad (2.37)$$

$$\text{Re}(\epsilon_r) = 1 + \frac{nq^2}{2m\omega\epsilon_0} \frac{(\omega_0 - \omega)}{(\omega_0 - \omega)^2 + \gamma^2/4} \quad (2.38)$$

$$\text{Im}(\epsilon_r) = -\frac{nq^2}{2m\epsilon_0} \frac{\gamma/2}{(\omega_0 - \omega)^2 + \gamma^2/4} \quad (2.39)$$

$$\text{If } \omega \rightarrow 0, \text{ then } \epsilon_r - 1 \rightarrow \frac{nq^2}{m\epsilon_0\omega_0^2} \quad (2.40)$$

$$\text{If } \omega \rightarrow \infty, \text{ then } \epsilon_r - 1 \rightarrow 0 \quad (2.41)$$

$$P(t) = P_0 \exp(-t/\tau) \quad (2.42)$$

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$$f(\omega) = A \int_0^{\infty} P(t) \exp(-i\omega t) dt \quad (2.43)$$

$$P(t) = P_0 \exp(-t/\tau) \quad (2.44)$$

$$\epsilon_r(\omega) = A \int_0^{\infty} P_0 \exp(-t(i\omega + 1/\tau)) dt \quad (2.45)$$

$$\epsilon_r(\omega) = \frac{AP_0}{1/\tau + i\omega} + B \quad (2.46)$$

$$\epsilon_r(0) = AP_0\tau + B \quad \text{and} \quad \epsilon_r(\infty) = B \quad (2.47)$$

$$\frac{\epsilon_r(\omega) - \epsilon_r(\infty)}{\epsilon_r(0) - \epsilon_r(\infty)} = \frac{1}{1 + i\omega\tau} \quad (2.48)$$

$$\text{Re}(\epsilon_r(\omega)) = \epsilon_r(\infty) + \frac{\epsilon_r(0) - \epsilon_r(\infty)}{1 + \omega^2\tau^2} \quad (2.49)$$

$$\text{Im}(\epsilon_r(\omega)) = \omega\tau \frac{\epsilon_r(0) - \epsilon_r(\infty)}{1 + \omega^2\tau^2} \quad (2.50)$$