Electrical and optical properties of materials

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Part 4: Maxwell's Equations

$$E = f(z - vt) \tag{4.1}$$

$$\frac{\partial E}{\partial z} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial z}, \qquad \text{and} \quad \frac{\partial u}{\partial z} = 1 \tag{4.2}$$

$$\frac{\partial^2 E}{\partial z^2} = \frac{\partial^2 f}{\partial u^2}\frac{\partial u}{\partial z} = \frac{\partial^2 f}{\partial u^2} \tag{4.3}$$

$$\frac{\partial E}{\partial t} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial t}, \qquad \text{here} \quad \frac{\partial u}{\partial t} = -v \tag{4.4}$$

$$\frac{\partial^2 E}{\partial t^2} = -v\frac{\partial^2 f}{\partial u^2}\frac{\partial u}{\partial t} = v^2\frac{\partial^2 f}{\partial u^2} \tag{4.5}$$

$$\frac{\partial^2 E}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 E}{\partial t^2} \tag{4.6}$$

$$E = E_0 \cos\left(\frac{2\pi}{\lambda}(z - vt)\right) \tag{4.7}$$

$$E = E_0 \cos\left(kz - 2\pi f t\right) \tag{4.8}$$

$$E = E_0 \cos\left(kz - \omega t\right) \tag{4.9}$$

$$k^2 E = \frac{1}{v^2}\omega^2 E \qquad \text{or} \qquad v = \omega/k \tag{4.10}$$

$$(E\delta x - (E + \delta E)\delta x) = \frac{\delta B_y}{\delta t}\delta x \delta z \tag{4.11}$$

$$\frac{\delta E_x}{\delta z} = -\frac{\delta B_y}{\delta t} \tag{4.12}$$

$$(B\delta y - (B + \delta B)\delta y) = \mu J_x \delta y \delta z \tag{4.13}$$

$$\frac{\delta B_y}{\delta z} = -\mu J_x \tag{4.14}$$

$$\frac{\partial B_y}{\partial z} = -\mu\left(J_x + \epsilon\frac{\partial E_x}{\partial t}\right) \tag{4.15}$$

$$\frac{\partial^2 E_x}{\partial z^2} = -\frac{\partial}{\partial z}\frac{\partial B_y}{\partial t} \tag{4.16}$$

$$\frac{\partial^2 E_x}{\partial z^2} = -\frac{\partial}{\partial t}\frac{\partial B_y}{\partial z} \tag{4.17}$$

$$\frac{\partial^2 E_x}{\partial z^2} = \frac{\partial}{\partial t}\left(\mu_0\epsilon_0\frac{\partial E_x}{\partial t}\right) \tag{4.18}$$

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0\epsilon_0\frac{\partial^2 E_x}{\partial t^2} \tag{4.19}$$

$$\frac{1}{v^2} = \mu_0\epsilon_0 \qquad \text{or} \quad v = \frac{1}{\sqrt{\mu_0\epsilon_0}} \tag{4.20}$$

$$\mu_0 = 4\pi\times 10^{-7}\ \mathrm{Hm^{-1}} \qquad \text{and} \qquad \epsilon_0 = 8.854\times 10^{-12}\ \mathrm{s^2H^{-1}m^{-1}} \tag{4.21}$$

$$c_0 = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 299\ 792\ 458\ \mathrm{ms^{-1}} \tag{4.22}$$

$$c = \frac{1}{\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}} = \frac{c_0}{\sqrt{\epsilon_r\mu_r}} \tag{4.23}$$

$$n = \frac{c_0}{c} = \sqrt{\epsilon_r\mu_r} \tag{4.24}$$

$$ikE_0\exp(i(kz-\omega t)) = i\omega B_0\exp(i(kz-\omega t)) \tag{4.25}$$

$$\frac{E_0}{B_0} = \frac{\omega}{k} = c \tag{4.26}$$

$$Z = \frac{E_x}{H_y} = \frac{\mu E_0}{B_0} = \mu c = \frac{\mu}{\sqrt{\mu\epsilon}} = \sqrt{\frac{\mu}{\epsilon}} \tag{4.27}$$

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\ \Omega \qquad (= 120\pi\ \Omega \text{ is a common approx.}) \tag{4.28}$$

$$Z = \mu_r\mu_0 c = \sqrt{\frac{\mu_0\mu_r}{\epsilon_0\epsilon_r}} \tag{4.29}$$

$$\frac{\partial^2 E_x}{\partial z^2} = -\frac{\partial}{\partial z}\frac{\partial B_y}{\partial t} \tag{4.30}$$

$$\frac{\partial^2 E_x}{\partial z^2} = -\frac{\partial}{\partial t}\frac{\partial B_y}{\partial z} \tag{4.31}$$

$$\frac{\partial^2 E_x}{\partial z^2} = \mu\frac{\partial}{\partial t}\left(J_x + \epsilon\frac{\partial E_x}{\partial t}\right) \tag{4.32}$$

$$\frac{\partial^2 E_x}{\partial z^2} = \mu\frac{\partial}{\partial t}\left(\sigma E_x + \epsilon\frac{\partial E_x}{\partial t}\right) \tag{4.33}$$

$$\frac{\partial^2 E_x}{\partial z^2} = \mu\sigma\frac{\partial E_x}{\partial t} + \mu\epsilon\frac{\partial^2 E_x}{\partial t^2} \tag{4.34}$$

$$k = \sqrt{\mu\epsilon\omega^2 + i\omega\mu\sigma} \tag{4.35}$$

$$E = E_0 e^{i(kz-\omega t)} = E_0 e^{i(\mathrm{Re}[k]z-\omega t)} e^{-\mathrm{Im}[k]z} \tag{4.36}$$

$$\mathrm{Re}[k] = \mathrm{Im}[k] = \sqrt{\frac{\sigma\mu\omega}{2}} \tag{4.37}$$

$$\delta = \sqrt{\frac{2}{\sigma\mu\omega}} \tag{4.38}$$

$$U = \frac{1}{2}(\mathbf{D}\cdot\mathbf{E} + \mathbf{B}\cdot\mathbf{H}) = \frac{1}{2}(\epsilon E_x^2 + \mu H_y^2), \tag{4.39}$$

$$cU = \frac{1}{2}\left(\sqrt{\frac{\epsilon}{\mu}}E_x^2 + \sqrt{\frac{\mu}{\epsilon}}H_y^2\right) = \frac{1}{2}\left(\frac{E_x^2}{Z} + ZH_y^2\right), \tag{4.40}$$

$$flux = cU = E_x H_y = \frac{E_x^2}{Z} = ZH_y^2 \tag{4.41}$$

$$\mathbf{P} = \mathbf{E}\times\mathbf{H} \qquad (P_z = E_x H_y) \tag{4.42}$$