Electrical and optical properties of materials

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Part 5: Optical properties of materials

$$k = \frac{\omega}{c} = \frac{\omega n}{c_0} = nk_0 \tag{5.1}$$

$$E_x = E_0 \exp\left[i\left(nk_0 z - \omega t\right)\right] \tag{5.2}$$

$$\begin{array}{lll} E(A) = E_A e^{i(n_1 k_0 z - \omega t)}, & E(B) = E_B e^{i(n_1 k_0 z - \omega t)}, & E(C) = E_C e^{i(n_2 k_0 z - \omega t)} \\ H(A) = \frac{E_A}{Z_1} e^{i(n_1 k_0 z - \omega t)}, & H(B) = -\frac{E_B}{Z_1} e^{i(n_1 k_0 z - \omega t)}, & H(C) = \frac{E_C}{Z_2} e^{i(n_2 k_0 z - \omega t)} \end{array} \tag{5.3}$$

$$E_A + E_B = E_C \qquad \text{and} \qquad \frac{E_A}{Z_1} - \frac{E_B}{Z_1} = \frac{E_C}{Z_2} \tag{5.4}$$

$$\text{reflected} \quad \frac{E_B}{E_A} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \tag{5.5}$$

$$\text{transmitted} \quad \frac{E_C}{E_A} = \frac{2Z_2}{Z_2 + Z_1} \tag{5.6}$$

$$\text{reflected} \quad \frac{E_B}{E_A} = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{n_1 - n_2}{n_1 + n_2} \tag{5.7}$$

$$\text{transmitted} \quad \frac{E_C}{E_A} = \frac{2Z_2}{Z_2 + Z_1} = \frac{2n_1}{n_1 + n_2} \tag{5.8}$$

$$E(A) = E_A \exp[i(n_1 k_0 \vec{a} - \omega t)] \tag{5.9}$$

$$E(B) = E_B \exp[i(n_1 k_0 \vec{b} - \omega t)] \tag{5.10}$$

$$E(C) = E_C \exp[i(n_2 k_0 \vec{c} - \omega t)] \tag{5.11}$$

$$\vec{a} = \cos\theta_a \vec{x} + \sin\theta_a \vec{y} \tag{5.12}$$

$$\vec{b} = -\cos\theta_b \vec{x} + \sin\theta_b \vec{y} \tag{5.13}$$

$$\vec{c} = \cos\theta_c \vec{x} + \sin\theta_c \vec{y} \tag{5.14}$$

$$i\left(n_1 k_0 \sin\theta_a \vec{y} - \omega t\right) \begin{aligned} &= i\left(n_1 k_0 \sin\theta_b \vec{y} - \omega t\right) \\ &= i\left(n_2 k_0 \sin\theta_c \vec{y} - \omega t\right) \end{aligned} \tag{5.15}$$

$$n_1 \sin\theta_a = n_1 \sin\theta_b \qquad \text{and so} \qquad \theta_b = \theta_a, \tag{5.16}$$

$$n_1 \sin\theta_a = n_2 \sin\theta_c \qquad \text{and so} \qquad \sin\theta_c = \frac{n_1}{n_2}\sin\theta_a \tag{5.17}$$

$$-\frac{E_B(x)}{E_A(x)} = \frac{E_B(y)}{E_A(y)} = \frac{\sin 2\theta_c - \sin 2\theta}{\sin 2\theta_c + \sin 2\theta} \qquad \text{reflected} \tag{5.18}$$

$$\frac{E_C(x)}{E_A(x)} = \frac{E_C(y)}{E_A(y)} = \frac{4\sin\theta_c\cos\theta}{\sin 2\theta_c + \sin 2\theta} \qquad \text{transmitted} \tag{5.19}$$

$$r_{ij} = \frac{n_i - n_j}{n_i + n_j} \qquad \text{and} \qquad t_{ij} = \frac{2n_i}{n_i + n_j} \tag{5.20}$$

$$r_{12} = \frac{n_1 - n_2}{n_1 + n_2} = r_{23} = \frac{n_2 - n_3}{n_2 + n_3} \tag{5.21}$$

$$\phi = 2Ln_2k_0 \tag{5.22}$$

$$2Ln_2k_0 = \pi \tag{5.23}$$

$$L = \frac{1}{n_2}\frac{\pi}{2k_0} = \frac{1}{n_2}\frac{\lambda_0}{4} = \frac{\lambda_2}{4}, \tag{5.24}$$

$$\phi_1 = \phi_2, \quad \phi_{2n+1} = \phi_{2n} + 2\pi \quad \text{and} \quad \phi_{2n+2} = \phi_{2n} + 2\pi \tag{5.25}$$

$$\phi_2 = \phi_1 + \pi, \quad \phi_{2n+1} = \phi_{2n} + 3\pi \quad \text{and} \quad \phi_{2n+2} = \phi_{2n} + 4\pi \tag{5.26}$$

$$\frac{n_1}{n_2}\sin\theta > 1 \tag{5.27}$$

$$\sin\theta_a \leq \frac{n_1}{n_2}, \qquad \text{with} \qquad \sin(90 - \theta_c) = \cos\theta_c = \frac{n_2}{n_1}\sin\theta_a \tag{5.28}$$

$$\sin\theta_c > \frac{n_2}{n_1} \tag{5.29}$$

$$\sin\theta_a < \sqrt{\left(\frac{n_1}{n_2}\right)^2 - 1} \tag{5.30}$$

$$n = \sqrt{\epsilon_r \mu_r} \sim \sqrt{\epsilon_r} \tag{5.31}$$

$$A_1 = A_0 \cos\theta \tag{5.32}$$

$$I_1 \propto A_1^2 = A_0^2 \cos^2\theta = I_0 \cos^2\theta \tag{5.33}$$

$$A_n = A_0 \cos^n \delta\theta \tag{5.34}$$

$$I_n = I_0 \cos^{2n} \delta\theta \tag{5.35}$$

$$I_n \approx I_0 \left(1 - \frac{\delta\theta^2}{2}\right)^{2n}, \tag{5.36}$$

$$I_n \approx I_0 I_0 \left(1 - n\delta\theta^2\right) \tag{5.37}$$

$$I_n = I_0 \left(1 - \frac{\phi^2}{n}\right) \tag{5.38}$$