Problems

Q.1 Ionic conductivity

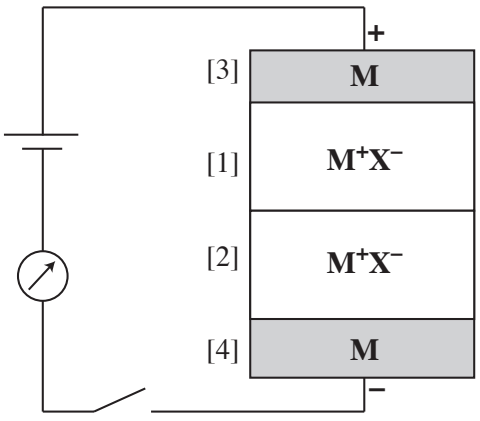
The conductivity σ of an alkali halide crystal containing a small amount of the same halide of a divalent metal varies with temperature T in the following way:

T/K	σ (Sm^{-1})	T/K	σ (Sm^{-1})
300	1.35×10^{-7}	800	4.39×10^{-2}
400	1.27×10^{-5}	900	2.09×10^{-1}
500	1.86×10^{-4}	1000	9.37×10^{-1}
600	1.20×10^{-3}	1100	2.97
700	6.95×10^{-3}		

Deduce as much as you can from these data.

Q.2

An experiment is set up as depicted in the figure below to deduce the relative contributions of Na^+ and Cl^- ions to the conductivity of NaCl.



After passing a current of 1 mA for 10 hours, the four pieces are separately weighed. Piece [2] has decreased in mass by 2.2 mg, while [1] has grown by 2.2 mg. Piece [3] has decreased in mass by 8.6 mg while [4] has grown by 8.6 mg. Find the relative contributions of Na^+ and Cl^- ions to the conductivity.

Q.3 Superconductivity

You have been given a large grant from a recent government stimulus package to design a system of levitating skateboards, for individual public transportation over short distance. The finance minister has shares in a large tin mine and so has required that you perform your initial design study using this material.

a) What type of superconductor is tin?

b) What is the critical current through a wire of tin with 1 mm radius at 2 K? The critical field of tin at 2 K is 20 mT.

Consider a small piece S of tin below T_c (with volume V and density ρ) which is placed directly at some height r above a wire with current I. The magnetostatic force on S is equal to:

$$F = m\frac{\partial B}{\partial r},$$

where m is the total magnetic moment(= volume $\times$ magnetisation M), and $\frac{\partial B}{\partial r}$ is the magnetic field gradient along r.

c) Calculate the magnetic field at S, the magnetic field gradient $\frac{\partial B}{\partial r}$, and the magnetic moment of S.

d) By equating the gravitational and magnetostatic forces on S, find a relation between the current through the wire I and the equilibrium height of S above the wire.

e) If the wire is made of tin and kept at 2 K, how high could one levitate S if it's density is 5000 kgm^{-3}.

f) Is this equilibrium position stable? If one had more freedom in design choices, what might you do, and why? Give as many details as you can think of.

Q.4 Dielectric materials

Discuss the physical mechanisms which contribute to the polarisability of dielectric materials.

Q.5

The static relative permittivity ϵ_r of CO_2 and NH_3 is measured at 273 K and 373 K at a pressure of 10^5 Pa (1 atm) and the values are found to be:

	CO_2	NH_3
273 K	1.000988	1.00834
373 K	1.000723	1.00487

Calculate the permanent electric dipole moment for each gas, and also the radius of the molecule, assuming the electronic polarisability to be the same as that of a conducting sphere ($\alpha = 4\pi\epsilon_0 a^3$).

Note: the volume of 1 mole of gas at 1 atm and 273 K is 22.4 litres.

Q.6

a) Estimate the dipole moment of a water molecule given that the static dielectric constant of water at room temperature is 80 and that the refractive index of water for visible light is 1.33.

b) The figure below plots measurements of the relative permittivity of water as a function of frequency and temperature (0° to 100°C; arrows show increasing temperature). Suggest how such a plot might have been obtained and explain the principle features. What mechanisms is responsible for the frequency and temperature dependence? Extract quantitative information on the physical properties of water. Microwave ovens typically operate at 2.45 GHz. Why might this be a suitable choice for heating food?